International Centre for Scientific Culture "E. Majorana"

School of Mathematics "G. Stampacchia"

" 77th VARIATIONAL ANALYSIS AND APPLICATIONS "

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ABSTRACTS OF LECTURES AND SHORT COMMUNICATIONS

Maria Colombo (EPFL, Lausanne)

Flessibilità dei flussi di Eulero bidimensionali

Per le equazioni di Eulero bidimensionali, un risultato classico di Yudovich afferma che le soluzioni sono uniche nella classe della vorticità limitata; è un celebre problema aperto se questo risultato di unicità possa essere esteso alla classe delle L^p-vorticità. Negli ultimi anni, molti contributi hanno portato a progressi significativi: solo per citarne alcuni, ci sono stati risultati basati sull'integrazione convessa, sull'instabilità in variabili autosimili, sullo studio di certi vortici puntuali, evidenze numeriche.
La conferenza fornirà una panoramica di questi sviluppi e metterà in evidenza i nuovi risultati ottenuti grazie al metodo di integrazione convessa.

Gianni Dal Maso (SISSA Trieste, IT)

Homogenisation of free discontinuity problems: the vectorial case

We study deterministic and stochastic homogenisation problems for free discontinuity functionals dependingon vector-valued functions under new hypotheses on the surface energies. The results are based on a compactness theorem with respect to Gamma-convergence, on the characterisation of the integrands of the Gamma-limit by means of limits of minimum values of some auxiliary minimum problems on small cubes, and on the subadditive ergodic theorem for the stochastic part.

Claudia Alejandra Sagastizabal (University of Campinas, BR)

Weak convexity and approximate subdifferentials

We explore and construct an enlarged subdifferential for weakly convex functions. The resulting object turns out to be continuous with respect to both the function argument and the enlargement parameter. We carefully analyze connections with other constructs in the literature and particularize to the weakly convex setting well-known variational principles. By resorting to the new enlarged subdifferential, we provide an algorithmic pattern of descent for weakly convex minimization. Under minimal assumptions, we show subsequential convergence to a critical point, and links with difference-of-convex algorithms and criticality conditions are also discussed. Joint work with Wim van Ackooij and Felipe Atenas.

Caroline Geiersbach (WIAS Berlin, DE)

Probabilistic state constraints for optimal control problems under uncertainty

In this talk, we discuss optimal control problems subject to random state constraints, where we distinguish between the chance-constrained case and the almost sure formulation. We highlight some of the difficulties in the infinite-dimensional setting, which is of interest in physics-based models where a control belonging to a Banach space acts on a system described by a partial differential equation (PDE) with random inputs or parameters. We study the setting in which the obtained state should be bounded uniformly over the physical domain with high probability, or even probability one. We apply our results to a model with a random elliptic PDE, where the randomness is induced by the right-hand side. For the chance-constrained setting, this structure allows us to obtain an explicit representation for the Clarke subdifferential of the probability function using the spherical radial decomposition of Gaussian random vectors. This representation is used for the numerical solution in a discretize-then-optimize approach. For the almost sure setting, we use a Moreau-Yosida regularization and solve a sequence of regularized problems in an optimize-then-discretize approach. The solutions are compared, providing insights for the development of further algorithms.

Russell Luke (Universitat Goettingen, DE)

Superlinearly convergent bundle-type methods for optimization with 0-dimensional partial smoothness

Bundle methods like the UV method of Mifflin and Sagastizabal are well-established for problems with nondegenerate semi-smooth or partially smooth structure. These methods only reach their full potential, however, when the partially smooth manifold on which solutions lie has nonzero dimension. We focus on the case of convex optimization problems with pencil-like structure at solutions. We present an approach that obtains superlinear convergence in this critical case and, under sufficient (but not surprising) regularity assumptions, d-step quadratic convergence.

Antonio Leaci (Università del Salento)

Symmetrised Fractional Variation with L^1 fidelity for signal and image denoising

In this talk we introduce and apply a variational model for signal and image analysis based on Riemann-Liouville fractional derivatives. Both the one-dimensional and two-dimensional cases are studied. The model exploits an L^1 fitting data term together with both right and left Riemann-Liouville fractional derivatives as regularizing terms: the aim is to achieve an orientation independent approach.
Some numerical experiments in one dimension for signal denoising are shown, by exploiting the Grünwald-Letnikov scheme and aiming to calibrate the fidelity parameter with the fractional order by white noise autocorrelation.
These results are obtained in a joint research project with Alessandro Lanza (Università di Bologna), Serena Morigi (Università di Bologna) and Franco Tomarelli (Politecnico di Milano).

Helene Frankowska (IMJ, Sorbonne University, Paris, FR)

Nonsmooth Analysis in Wasserstein Spaces and some Applications

Dynamical systems involving large number of agents can be approached by considering moving sets of agents instead of union of vector-valued time-dependent paths. In some social sciences models, like the evacuation one, it may be interesting to assign to sets their measures and to work with metric spaces of Borel probability measures, the so called Wasserstein spaces. The dynamics of measures are described then either via continuity (transport) equations or continuity inclusions. The questions like invariance of sets under continuity inclusions can be considered in this framework. Actually, classical invariance results known in the Euclidean spaces have their analogues in the Wasserstein spaces. For this aim the notions of tangents, normals and proximal normals have to be introduced in the context of (metric) Wasserstein spaces. Here translation of vectors is replaced by the push-forward operation # on measures via measurable mappings. This leads to a natural definition of tangent cone to a subset Q of the Wassestein space at $\mu \in Q$ as a subset of all Borel measurable functions f with $(Id+\epsilon f)\#\mu$ infinitesimally close to Q with respect to ϵ. Then normals are again functions satisfying appropriate integral inequalities. Proximal normals to Q are defined via barycentric projections of transport plans between a measure and its projection on Q. In this talk I will present necessary and sufficient conditions from [2] and [3] for the existence of solutions to state-constrained continuity inclusions. They were applied in [3] to investigate uniqueness of solutions to HJB equations and will be recalled by the end of the talk.

[1] Bonnet B. and Frankowska H., Carathéodory Theory and a Priori Estimates for Continuity Inclusions in the Space of Probability Measures, Nonlinear Analysis, 2024.

[2] Bonnet B. and Frankowska H., On the Viability and Invariance of Proper Sets under Continuity Inclusions in Wasserstein Spaces, SIAM Journal on Mathematical Analysis, 2024.

[3] Badreddine Z. and Frankowska H., Solutions to Hamilton-Jacobi equation on a Wasserstein space, Calculus of Variations and PDEs, 81:9, 2022.

Hasnaa Zidani (INSA Rouen, FR)

Hamilton-Jacobi approach for some optimal control problems in Cat(0) spaces

In this talk, we will explore a class of optimal control problems within the framework of Hadamard spaces, which are a type of non-positively curved metric spaces. Our focus will be on employing the Hamilton-Jacobi formalism to analyze these problems, providing a generalised notion of viscosity solutions to understanding the caracterization of the value function in such non-Euclidean settings. Additionally, we will present some illustrative examples that demonstrate the application of this theory to specific control problems in Hadamard spaces.

Martin Krûzík
(Institute of Information Theory and Automation of the Czech Academy of Sciences, CZ)

Non-interpenetration of matter in lower-dimensional structures

Non-interpenetration of matter is a well-known challenge for solid elastic materials combining analytical and geometrical aspects. In the bulk model, at least on the conceptual level, non-interpenetration is quite understood even if many challenges still remain open. In lower-dimensional structures (plates, rods), the situation seems to be even less clear. Focusing on rods in the plane, we will introduce a possible concept of noninterpenetration and show a density and Γ-limit result.
This is a joint work iwith B. Benešová, D. Campbell, and S. Hencl (all from Prague).

Serena MORIGI (Università di Bologna, IT)

Nonlinear compressed sensing with applications to ill-posed inverse problems

A significant portion of research focuses on linear Compressive Sensing (CS) problems and holds for well-posed CS models. However many real-world applications in physics and biomedical sciences involve inherent nonlinearities. In these cases, the linear model becomes inadequate. We propose a proof of concept study in using CS techniques in the numerical solution of nonlinear ill-posed inverse problems. In particular a sparsity-aware model for the solution of a variational formulation of the Electrical Impedence Tomography inverse problem.

William M. McEneaney (Univ. of California San Diego, US)

Staticization-Based Numerical Approach to the Schrödinger Equation

We first develop a solution representation for the Schrödinger initial value prob-

lem in the case of a Coulomb potential. The representation is in terms of the expectation of a payoff over complex-valued diffusion processes, where existence of solutions to the stochastic differential equation defining the diffusion was previously established. Of particular interest here are representations in the spirit of the Feynman path-integral interpretation, where in particular, one looks at a certain action-based functional, S, where the solution of the Schrödinger initial value problem is then obtained through the use of Maslov quantization/dequantization.
Recent efforts on the use of stationary action for solution of two-point boundary value problems in conservative systems has led to the development of the staticization operator, "stat", which maps functions into their values at stationary (i.e., critical) points. Notably, staticization generalizes optimization in the case of differentiable functions. Specifically, although the domains of optimization operators are restricted to functions with range in the reals, the stat operator may be applied to differentiable functions with ranges in Banach spaces, notably including the case of the complex plane.
The above representation for S is as the staticization of the expectation of the integral of a complex-valued action functional, where the staticization is over a velocity control. Using a staticization-based representation for an extension of the Coulomb potential to complex-valued space, one obtains a representation for S in terms of an iterated pair of stat operations on the expectation. Through recent results regarding iterated stat operators, the ordering of the operators is inverted. After the reordering, the inner problem is in a linear-quadratic-Gaussian form, leading to solution as a quadratic function, with coefficients generated by differential Riccati equations. The solution of the original problem is then obtained by staticization over these quadratic functions. A discretized approximation of the stat operator is employed, resulting in an efficient numerical method.
The presentation relies also on the efforts of Prof. H. Kaise, Prof. P.M. Dower and Dr. R. Zhao.

Matteo Novaga **(Dipartimento di Matematica, Pisa, IT)**

Periodic partitions with minimal perimeter

I will discuss existence, regularity and qualitative properties of fundamental domains which minimize the classical or the fractional perimeter, under volume contraint or with fixed inradius.

Edoardo Mainini **(Università degli studi di Genova, IT)**

Linearization of finite elasticity with surface tension

We discuss linearization of finite elasticity by means of Γ-convergence in presence of surface energy terms, for both place and traction problems.

Daniel Walter (Humboldt-Universität zu Berlin, DE)

On the extremal points associated to some vectorial total variation seminorms

A precise characterization of the extremal points of sublevel sets of nonsmooth penalties provides both detailed information about minimizers, and optimality conditions in general classes of minimization problems involving them. Moreover, it enables the application of accelerated generalized conditional gradient methods for their efficient solution
In this talk, we discuss the extremal points induced by gradient total variation seminorms for vector-valued functions on bounded Euclidean domains. On the one hand, for not fully vectorial cases in which either the domain or the target are one dimensional we prove that these extremals are fully characterized as in the scalar-valued case, that is, they consist of piecewise constant functions with two regions. On the other, in more complicated settings, e.g. for spectral matrix norms, we produce families of examples to show that the resulting set of extremal points is larger and includes piecewise constant functions with more than two regions. Finally, as an example of an extremal which is not piecewise constant, we prove that unit radial vector fields are extremal for the Frobenius total variation in the plane.
The talk is finished by discussing similar results for the total deformation induced by the symmetrized gradient. For this case, we show piecewise infinitesimally rigid functions with two pieces to be extremal under mild assumptions. This is joint work with Kristian Bredies and José A. Iglesias.

Alberto Roncoroni (Politecnico di Milano, IT)

Stable minimal hypersurfaces in $\mathbb{R}^4$

A celebrated result by S.N. Bernstein asserts that the planes are the only entire minimal (i.e. critical points of the area functional) graphs in $\mathbb{R}^3$. This result, known as the Bernstein problem, has been generalized for entire minimal graphs in $\mathbb{R}^{n+1}$, with $n \leq 7$ and it turns out to be false for $n \geq 8$. Motivated by the fact that minimal graphs are actually stable (i.e. the second variation of the area functional is non-negative), the following natural generalization of the Bernstein problem is still an open and fascinating question: if M is a complete, orientable, immersed, stable, minimal hypersurface in $\mathbb{R}^{n+1}$, does it have to be necessarily a hyperplane? In a recent paper Chodosh and Li proved that this is true in $\mathbb{R}^4$. In this talk I will discuss an alternative proof of the result by Chodosh and Li obtained in collaboration with G. Catino and P. Mastrolia.

Adriana Garroni (La Sapienza, Roma, IT)

Asymptotics of a discrete model for partial defects

I will present a discrete two dimensional toy model for crystal defects based on nearest neighbours and next to nearest neighbours interaction, via a period potentials. In the asymptotic limit as the lattice spacing tends to zero, in terms of Gamma convergence, the model accounts for the formation and interaction of partial dislocations and stacking faults. The model falls into a class of models with topological (fractional) singularities. One of the key ingredients is the characterisation of the minimisers for the core energy of the singularities, the so called one-vortex solutions.

Aris Danilidis (VADOR, TU Wien)

Everywhere differentiable Lipschitz continuous functions can be highly pathological

The talk sheds light on the difference between differentiable vs strict differentiable Lipschitz functions from the view point of nonsmooth analysis: while in the latter class, the limiting subdifferential is always reduced to a singleton, the limiting subdifferential of a differentiable Lipschitz function may assume almost every possible value. A concrete example-scheme will be presented revealing that the class of such pathological locally Lipschitz differentiable functions is dense (for the topology of the uniform convergence) and spaceable (for the Lip-norm topology). As a by-product, we obtain the following surprising result: all convex bodies of a finite dimensional space are contained in the range of the subdifferential of some real-valued differentiable locally Lipschitz function.
This is a joint work with R. Deville (Bordeaux) and S. Tapia (Vienna)

Giovanni Colombo (Università di Padova, IT)

Recent advances on the optimal control of Moreau's sweeping process

The sweeping process is a differential inclusion of dissipative type that occurs in the modeling of several phenomena, including contact mechanics, the simplest example being the "play operator" that occurs in some models of hysteresis. This differential inclusion describes the movement of a particle that is constrained in a moving (convex or mildly nonconvex) set C(t) and is such that its velocity is an (inward) normal to the boundary of C(t). Since the Seventies, several results on existence of solutions were obtained. More recently, some studies on the optimal control of the sweeping process were also performed. The talk aims at illustrating some results on necessary optimality conditions for the sweeping process with a controlled drift, namely the differential inclusion $x'(t) \in N_C(t)(x(t)) + f(x(t), u(t)); x(0) = x_0 \in C(0)$. Here x is the state, that belongs to a finite dimensional space, C is a moving set that is either a polyhedron (whose faces can be controlled as well) or an intersection of finitely many smooth constraints, and u is the control. Two versions of necessary optimality conditions of the type of Pontryagin Maximum Principle will be presented.

Mikhail V. Solodov (IMPA, BR)

Descent sequences in weakly convex optimization

We present a framework for analyzing convergence and local rates of convergence of a class of descent algorithms, assuming the objective function is weakly convex. The framework is general, in the sense that it combines the possibility of explicit iterations (based on the gradient or a subgradient at the current iterate), implicit iterations (using a subgradient at the next iteration, like in the proximal schemes), as well as iterations when the associated subgradient is specially constructed and does not correspond neither to the current nor the next point (this is the case of descent steps in bundle methods). Under the subdifferential-based error bound on the distance to critical points, linear rates of convergence are established. Our analysis applies, among other techniques, to prox-descent for decomposable functions, the proximal-gradient method for a sum of functions, redistributed bundle methods, and a class of algorithms that can be cast in the feasible descent framework for constrained optimization.

Teemu Pennanen **(King's College, London)**

Convex stochastic optimization

We study dynamic programming, duality and optimality conditions in a general convex stochastic optimization format that extends earlier formulations e.g. in operations research, optimal control and financial mathematics. We establish the validity of dynamic programming, the existence of primal solutions and the absence of duality gap without compactness or boundedness assumptions. In the context of financial mathematics, the relaxed assumptions are satisfied under the well-known no-arbitrage condition and the reasonable asymptotic elasticity condition of the utility function. Besides financial mathematics, we obtain several new results in stochastic programming and stochastic optimal control. The existence of dual solutions is established in the general format under conditions that extend those given by Rockafellar and Wets for stochastic problems of Bolza.

Ludovic Rifford **(Cote d'Azur University, Nice, FR)**

On the minimizing Sard Conjecture in sub-Riemannian geometry

On the minimizing Sard Conjecture in sub-Riemannian geometryAfter recalling the notions of minimizing geodesics and singular horizontal curves in sub-Riemannian geometry, we will discuss various versions of the so-called Sard conjecture and present several result dealing with the minimizing Sard Conjecture. The proof of our main result will be sketched, it relies on tools from non-smooth analysis and geometric measure theory.

Marco Caroccia **(Politecnico di Milano, IT)**

On the singular planar Plateau problem

The classical Plateau problem asks which surface in three-dimensional space spans the least area among all the surfaces with boundary given by an assigned curve S. This problem has many variants and generalizations, along with (partial) answers, and has inspired numerous new ideas and techniques. This talk briefly introduce the problem in both its classical and modern contexts, and then we will focus on a specific vectorial (planar) type of the Plateau problem. Given a curve S in the plane, we can ask which diffeomorphism T of the disk D maps the boundary of D to S and spans the least area, computed as the integral of the Jacobian of T, among competitors with the same boundary condition. For simply connected curves, the answer is provided by the Riemann map, and the minimal area achieved is the Lebesgue measure of the region enclosed by S. For more complex curves, possibly self-intersecting, new analysis is required. I will present a recent result in this sense, obtained in collaboration with Prof. Riccardo Scala from the University of Siena, where the value of the minimum area is computed with an explicit formula that depends on the topology of S.

Andrea Natale **(Inria and LPP, Lille University, FR)**

Metric extrapolation in the Wasserstein space

In this talk, we study a variational problem providing a way to extend for all times minimizing geodesics connecting two given probability measures in the Wasserstein space. This is simply obtained by allowing for negative coefficients in the classical variational characterization of Wasserstein barycenters. We show that this problem admits two equivalent convex formulations: the first can be seen as a particular instance of Toland duality and the second is a barycentric optimal transport problem. Finally, we will show how this probem can be used to construct a higher order generalization of the JKO scheme for Wassersteing gradient flows. This is based on a joint work with T. O. Gallouët and G. Todeschi.

Antonella Nastasi **(Università degli studi di Palermo, IT))**

Variational methods for double phase functionals

The talk shall focus on a class of double phase integrals characterized by nonstandard growth conditions, which constitute an important sub-field of the calculus of variations. Variational methods are powerful tools in investigating the behaviour and regularity properties of minimizers and, more generally, quasiminimizers that minimize the energy integral up to a multiplicative constant.
The study focuses on regularity theory, specifically local and global higher integrability, for quasiminimizers of a double phase integral with (p, q)-growth.
The proofs are obtained working purely on a variational level, in the setting of metric measure spaces with a doubling measure and a Poincaré inequality. The main novelty is an intrinsic approach to double phase Sobolev-Poincaré inequalities.
The results were obtained in collaboration with Juha Kinnunen (Aalto University) and Cintia Pacchiano Camacho (Calgary University).

Sylvain Sorin (IMJ-PRG, Sorbonne University, Paris, FR)

Asymptotic value of repeated games: a variational approach

We consider two person zero-sum repeated games. 1) After recalling the framework and the notions of uniform and asymptotic values we will concentrate on the later and describe tools related to viscosity solutions. 2) A first approach is a direct link with differential games: weak approachability (Vieille) and dual game with incomplete information (Laraki). 3) A second tool uses variational inequalities and comparison properties for games with incomplete information (Cardaliaguet, Laraki and Sorin). 4) Recent results on non convergence for stochastic games (Vigeral, Ziliotto) lead to the study of games with vanishing stage duration (Neyman, Sorin and Vigeral). We will describe the link with differential games under several information structures and the connection with HJB equations on the space of measures.

Antonin Chambolle (CEREMADE, CNRS, Paris-Dauphine PSL)

Discrete to continuous crystalline curvature flow

In this joint work with Daniele DeGennaro (CEREMADE, Parma) and Massimiliano Morini (Parma) we study a fully space and time discrete implicit approximation of the curvature flow, for a surface tension defined by pairwise interactions on the discrete lattice (with bounded range). We study the convergence as the space and time steps go to zero (with different possible regime) and find, surprisingly, that in some cases we obtain the limiting crystalline curvature flow in any convergence regime.
